1. Consider the surface $S$ in $\mathbb{E}^3$ defined by the equation $x^2 + y^3 = 9 - z^2$.

(a) Express $S$ as an isoset of some scalar field $f(x, y, z)$, and use this to find a vector field that is normal to $S$ at each point.

(b) Find normal form of the plane tangent to $S$ at the point $(2, 1, 2)$.

(c) Find parametric form of the line perpendicular to $S$ at the point $(2, 1, 2)$.

(d) Let $\gamma$ be the parametric curve $(x, y, z) = (\cos t, 2, \sin t)$, $t : 0 \rightarrow 2\pi$.
Show that $\gamma$ lies on the surface $S$.

2. Find the flux of the vector field $\vec{F}(x, y, z) = \langle x \sin^2 z + \arctan(y + z), y \cos^2 z - 10xz, x + y - 3z \rangle$ through the unit sphere in $\mathbb{E}^3$. [Be sure to indicate the direction in which the flux is going!]

3. Find the circulation of the vector field $\vec{F}(x, y, z) = \langle x^2 + y^2 + e^x, 4x + \cos y, 2xyz + \cos z \rangle$ clockwise (viewed from above) around the triangle in the $xy$-plane having vertices $(0, 0, 0)$, $(2, 0, 0)$ and $(0, 2, 0)$.

4. Consider the vector field $\vec{F}(x, y, z) = \langle 2x - y + \sin(x + z), ze^y + 3y - x, \sin(x + z) + e^y \rangle$.

(a) Find a potential function $f(x, y, z)$ for $\vec{F}$.

(b) Find the circulation of $\vec{F}$ along the path $(x, y, z) = (2t - 1, \cos \pi t, t^4 + t)$, $t : 0 \rightarrow 1$.

(c) Find the circulation of $\vec{F}$ around the unit circle in the $xy$-plane.

(d) Suppose that $\vec{G}(x, y, z) = \langle 2x + y, y + z, x + y + z \rangle$. Is $\vec{G}$ conservative? Why or why not?

5. If $f(x, y, z) = x^3 \ln(1 + z^2) + \cos(y + 2\sin x) - \frac{\arctan xyz}{e^{xyz}}$, find $\text{curl}(\nabla f)$.

6. Define a scalar field $f$ on $\mathbb{E}^2$ by $f(x, y) = x^2 - y^2 + xy + 1$.

(a) Show that $f$ is harmonic.

(b) If $R$ is a region in $\mathbb{E}^2$, show that the flux of the vector field $\nabla f$ through $\partial R$ is zero.

(c) Suppose $g$ is a smooth scalar field on $\mathbb{E}^2$ and that the outward flux of $\nabla g$ through the unit circle is $-2\pi$. Is it possible that $g$ is harmonic? Why or why not?

7. Suppose that $\vec{F}$ is a smooth irrotational vector field defined at every point of $\mathbb{E}^3$ except those on a theta-shape, as drawn on the attached page. Given that the circulation of $\vec{F}$ along the path $K$ is 3 and the circulation of $\vec{F}$ along the path $L$ is $-2$, use Stokes’ Theorem to find the circulation of $\vec{F}$ along each path drawn. For each part, indicate what surface you’re using and what normal you’ve chosen, and show the equation and computation that justifies your answer.