M&T §1.4

3. (a) \([1, 45^\circ, 1]\) sph \(\rightarrow \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 1\right)\) sph

\(\rightarrow \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 1\right)\) sph

• \([2, \frac{\pi}{4}, 4]\) sph \(\rightarrow (0, 2, -4)\) sph

\(\rightarrow (0, 2, -4)\) sph

• \([0, 90^\circ, 0]\) sph \(\rightarrow \left(0, 0, 10\right)\) sph

\(\rightarrow (0, 0, 10)\) sph

• \([3, \frac{\pi}{4}, 2]\) sph \(\rightarrow \left(\frac{\sqrt{2}}{\sqrt{2}}, \frac{\sqrt{2}}{2}, 2\right)\) sph

\(\rightarrow \left(\frac{\sqrt{2}}{\sqrt{2}}, \frac{\sqrt{2}}{2}, 2\right)\) sph

• \([4, \frac{\pi}{4}, 0]\) sph \(\rightarrow (1, 1, 0)\) sph

\(\rightarrow (1, 1, 0)\) sph

• \([2, \frac{3\pi}{4}, -2]\) sph \(\rightarrow (2, \frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{4})\) sph

\(\rightarrow (2, \frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{4})\) sph

(b) \((2, 1, -2)\) sph \(\rightarrow \left[3, \arctan \frac{1}{2}, \arccos \left(-\frac{3}{2}\right)\right]\) sph

\(\rightarrow \left[3, \arctan \frac{1}{2}, \arccos \left(-\frac{3}{2}\right)\right]\) sph

• \((0, 3, 4)\) sph \(\rightarrow \left[2, \frac{\pi}{4}, \frac{\pi}{2}\right]\) sph

\(\rightarrow \left[2, \frac{\pi}{4}, \frac{\pi}{2}\right]\) sph

• \((2, 0, 3)\) sph \(\rightarrow \left[\sqrt{3}, \arctan \frac{1}{\sqrt{3}}, \frac{\pi}{3}\right]\) sph

\(\rightarrow \left[\sqrt{3}, \arctan \frac{1}{\sqrt{3}}, \frac{\pi}{3}\right]\) sph

• \((-2, 1, -2)\) sph \(\rightarrow \left[2, \frac{\pi}{4}, \frac{\pi}{3}\right]\) sph

\(\rightarrow \left[2, \frac{\pi}{4}, \frac{\pi}{3}\right]\) sph


4. (a) \((r, \theta, z)\) sph \(\rightarrow \left(r, \theta, 2\pi - z\right)\) sph

(Make sure that this mapping is not the same as "turning space upside-down," the reflection indicated here reverses orientation: i.e., switching left-handed to right-handed — and is not physically realizable.)

(b) \((r, \theta, z)\) sph \(\rightarrow \left(r, \theta + \pi, -z\right)\) sph

(Make sure that this mapping is not the same as "turning space upside-down," the reflection indicated here reverses orientation: i.e., switching left-handed to right-handed — and is not physically realizable.)

(c) \((r, \theta, z)\) sph \(\rightarrow (-r, \theta, \pi - z)\) sph

(Make sure that this mapping is not the same as "turning space upside-down," the reflection indicated here reverses orientation: i.e., switching left-handed to right-handed — and is not physically realizable.)

5. (a) \((r, \theta, z)\) sph \(\rightarrow \left(r, \theta + \pi, z\right)\) sph: Rotate by 180° about the z-axis.

(b) \((r, \theta, z)\) sph \(\rightarrow \left(r, \theta + \pi, -z\right)\) sph: Reflect in the z-axis.

(Note that \(r \neq 0\) \(\Leftrightarrow \theta = \pi, \phi = \frac{\pi}{2} \Leftrightarrow z = 0\).

In general, \(z = \rho \cos \phi \Leftrightarrow \rho \cos(\pi - \phi) = -\rho \cos \phi = -z\).

(c) \((r, \theta, z)\) sph \(\rightarrow \left(2r, \theta + \frac{\pi}{2}, z\right)\) sph: Push all points twice as far from the origin \((\rho = 2\rho)\) and rotate right-handed by \(\pi\) radians \((90°)\) about the z-axis.

8. (a) \(n = n_o\): Cylinder of radius \(n_o\) around the z-axis

• \(\Theta = \Theta_o\): Radial (half-) plane emanating from the z-axis at a right-handed angle of \(\Theta_o\) from the x-axis.

• \(\Sigma = \Sigma_o\): Horizontal plane at position \(\Sigma_o\) along the z-axis.

(b) \(p = p_o\): Sphere of radius \(p_o\) centered at the origin

• \(\Theta = \Theta_o\): (Same as in cylindrical coordinates)

• \(\Phi = \Phi_o\): (Half-) cone with vertex at the origin and the z-axis as its axis, making an angle of \(\Phi_o\) with the z-axis.

18. E.g., point the z-axis along the axis of the cylinder, x-axis horizontal, y-axis down. Then the water corresponds to:

\[
\begin{align*}
0 & \leq r < 5 & \text{(Height)} \\
0 & \leq \phi < 10 & \text{(Radius)} \\
0 & \leq \theta < \frac{\pi}{2} & \text{(Half of 2\pi)}
\end{align*}
\]