The surface is given by the equation: $4y + 4z = 0$. To find the normal vector to the surface plane:

$$\frac{x}{4} + \frac{y}{4} = 0$$

At this point, $4\langle 0, -4, 0 \rangle + 4\langle 0, 0, 0 \rangle = 0 - 4\langle 0, -4, 0 \rangle$

Use the cross product of the component vectors. In this case, we can use A × B. The cross product gives a vector as a result. At this surface, the cross product is:

$$\langle -4, 4, 0 \rangle \times \langle 0, -4, 0 \rangle = \langle 0, 0, 0 \rangle$$

If $\mathbf{v} = \mathbf{u} \times \mathbf{w}$, then $\mathbf{v}$ is a skew symmetric matrix, then $\mathbf{v}$ is given by:

$$\mathbf{v} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

To find a unit normal vector $\langle 1, 1, 1 \rangle$, the surface lies in a plane. The plane lies at some plane, we need a point.

Applications: To find the plane number to a parametric vector.

Vectors & the Linear System. If $a, b, c$ are columns, $a + b + c = 0$. To find a unit normal vector $\langle 1, 1, 1 \rangle$. The plane lies at some plane, we need a point.
2. **Plane tangent to** $(x, y, z) = (u^2 - v^2, uv, u^2 + 4v)$ at $\left(-\frac{1}{2}, \frac{1}{2}, 2\right)$:

Our mapping is $(u, v) \rightarrow (u^2, uv, u^2 + 4v)$.

So $D\mathbf{F} = \begin{pmatrix} u & v \\ v & u^2 + 4v \\ 2u & 2uv \end{pmatrix}$. Crossing the columns:

$\begin{vmatrix} u & v \\ v & u^2 + 4v \\ 2u & 2uv \end{vmatrix} = (4-2u, -4uv - 8u, 2u + 2v)

Normal vector, for any $u, v$

$\begin{pmatrix} x & y \\ u & v \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$

$\begin{pmatrix} 4 & 0 \\ -7 & 1 \end{pmatrix}$ vector @ $(\frac{1}{2}, \frac{1}{2}, 2)$

At this point, $(4-2u, -4uv - 8u, 2u + 2v) = \langle 4, 0, 1 \rangle

Normal form to write the tangent plane:

$\langle P - P_0 \rangle \cdot \mathbf{n} = 0 \rightarrow \langle (x, y, z) - (-\frac{1}{2}, \frac{1}{2}, 2) \rangle \cdot \langle 4, 0, 1 \rangle = 0$

$4(x + \frac{1}{2}) + z - 2 = 0$

$\langle 4 - 2u, -4uv - 8u, 2u + 2v \rangle = \langle 0, 0, 0 \rangle$

$\Rightarrow 4 - 2u = 0, \quad 2u + 2v = 0, \quad -4uv - 8u = 0$

$v = 0 \quad \Rightarrow z = 4(u^2 - 1) - 8(0) = 16 - 16 = 0$

This surface is normal at all $(u, v) \neq (2, -2)$

9. **Unit normal to the parametrized surface** $\mathbf{x} = \left[0, \pi x, 0, \pi y \right] \rightarrow \mathbb{R}^3$

$\mathbf{x}(u, v) = (\cos \gamma \sin u, \sin \gamma \sin u, \cos \gamma)$

$D\mathbf{x} = \begin{pmatrix} \cos \gamma \sin u & -\sin \gamma \sin v \\ \sin \gamma \sin u & \cos \gamma \cos v \\ 0 & 0 \end{pmatrix}$

A normal $\mathbf{B}$ is given by the cross-product of the two columns:

$\begin{vmatrix} \cos \gamma \sin u & -\sin \gamma \sin v \\ \sin \gamma \sin u & \cos \gamma \cos v \\ 0 & 0 \end{vmatrix} = \left(0, \cos \gamma \sin u, + \sin \gamma \sin v \cos u \right)$

$||\mathbf{B}|| = \sqrt{\cos^2 \gamma \sin^2 u + \sin^2 \gamma \sin^2 v + \sin^2 \gamma \cos^2 u}$

$= \left(\sin^2 u + \sin^2 v \cos^2 u \right)^{1/2}$

$= \left(\sin^2 u + \sin^2 v \cos^2 u \right)^{1/2} = \left(\sin^2 u \right)^{1/2} = \sin u = \sin \gamma \left(\cos 0 \rightarrow \gamma \leq 0\right)$

$\mathbf{B}$ is the unit normal is $\mathbf{B} = \left(1, \cos \gamma \sin u, \sin \gamma \sin v \cos u \right)$

(technically, we've divided by zero when $u = 0$ or $\pi$ ... but if we think of the limit as $u \rightarrow 0$ or $u \rightarrow \pi$, this will be the correct value.)

$\mathbf{x}$ is just the spherical coordinate mapping, with $(u, v) \leftrightarrow (\phi, \theta) + p = 1$.

$\therefore$ this surface is the unit sphere in $\mathbb{R}^3$. 
11. UNIT NORMAL TO THE PARAMETRIC SURFACE $S : [0, 2\pi] \times [-1, 3]$,

$\mathbf{E}(u,v) = (\sin v, u, \cos v)$:

$\mathbf{E} \leftrightarrow x \begin{bmatrix} u \\ \cos v \\ 0 \end{bmatrix} \quad \text{So} \quad \mathbf{E} = \begin{bmatrix} \hat{u} \\ \hat{v} \\ \hat{z} \end{bmatrix} = \langle -\sin v, 0, -\cos v \rangle$

$\|\mathbf{E}\| = (\sin^2 v + \cos^2 v)^{1/2} = 1$, so $\mathbf{E}$ is already a unit vector.

\[ \text{UNIT NORMAL: } \langle -\sin v, 0, -\cos v \rangle \]

THE SURFACE IS A CYLINDER ALONG THE $y$-AXIS WITH UNIT RADIUS, RUNNING FROM $y = -1$ TO $y = 3$; THE EASIEST WAY TO SEE THIS IS TO LOOK AT $x \times z$. FIRST, WHICH TRACE OF THE UNIT CIRCLE IN THE $xz$-PLANE ($x = \sin v, z = \cos v$). THEN $y = u$ SIMPLY RANGES FROM $-1$ TO $3$, MAKING THIS INTO A CYLINDER.

(OR, RECOGNIZE THE SIMILARITY TO CYLINDRICAL COORDINATES WITH $r = 1$ AND THE AXES SWITCHED AROUND.)