1. Define the function $T : \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$ by $(\vec{a}, \vec{b}, \vec{c}) \mapsto \vec{a} \cdot (\vec{b} \times \vec{c})$; this function is commonly called the **triple product** of $\vec{a}, \vec{b},$ and $\vec{c}$.

[Be careful to note that the symbol “×” in the domain is used to mean “Cartesian product” for sets and “cross product” for vectors—but the meaning of the symbol should be clear from context.]

(a) Use the fact that · and × are bilinear to prove that $T$ is **trilinear**.

(b) Show that $T(\vec{a}, \vec{a}, \vec{b}), T(\vec{a}, \vec{b}, \vec{a}),$ and $T(\vec{b}, \vec{a}, \vec{a})$ all evaluate to zero, using the definitions and properties of · and ×. What adjective describes this property of $T$?

(c) Suppose that $(\vec{e}_1, \vec{e}_2, \vec{e}_3)$ is a right-handed orthonormal basis for $\mathbb{R}^3$. Use this information, along with the geometric definitions of · and ×, to find the value of $T(\vec{e}_1, \vec{e}_2, \vec{e}_3)$.

(d) In light of your results from parts (a–c), you should recognize that the triple product $T$ is the same function as another one we’ve already introduced—what is it?

2. Suppose that you have two distinct points $p_0, p_1 \in \mathbb{E}^n$.
   Describe the line through these two points in parametric form.

3. Working in $\mathbb{E}^3$:
   (a) Given a plane $P$ described in parametric form by $p = p_0 + t\vec{v} + s\vec{w}$ ($s, t \in \mathbb{R}$), express $P$ in normal form.

   (*b) Suppose that you start with a plane described in normal form $(p - p_0) \cdot \vec{N} = 0$. Translating this to parametric form isn’t as straightforward (try it). Can you think of any general concepts that help to explain why this is the case?

4. Working in $\mathbb{E}^3$, a line $p = p_0 + t\vec{v}$ and a point $q_0$ not on that line determine a plane $P$.
   (a) Describe $P$ in normal form.
   (b) Describe $P$ in parametric form.

   (*c) Can you explain why (in contrast to the case in the previous problem) we were able to easily express this plane in both forms?

5. Suppose that $p_0, p_1,$ and $p_2$ are three noncollinear points in $\mathbb{E}^3$, forming a triangle $T$.
   (a) Find the area of $T$. What does the fact that the three points are noncollinear tell you about this value?
   (b) Find the angles of the triangle $T$.
   (c) Find the plane containing $T$, expressed both in parametric form and in normal form.
   (d) Find the “center of mass of $T$” (using well-defined mathematical operations).
   (e) Find a parametric equation for the line perpendicular to $T$ and passing through the point $q_0$.
   (*f) Find the distance from the point $p_2$ to the line through $p_0$ and $p_1$.

6. In $\mathbb{E}^3$, suppose that two non-parallel planes $P$ and $Q$ intersect in a line $L$.
   If the planes are given in normal form by $(p - p_0) \cdot \vec{N} = 0$ and $(q - q_0) \cdot \vec{M} = 0$:
   (a) Find a vector parallel to $L$.

   (*b) [Difficult] Find a point on the line $L$, and verify that it satisfies the equations of both planes.